

# Trajectory Planning and Four-Leg Coordination for Stair Climbing in a Quadruped Robot

Chih-Chung Ko, Shen-Chiang Chen, Cheng-Hsin Li, and Pei-Chun Lin

**Abstract**—We report on the algorithm of trajectory planning and four leg coordination for quasi-static stair climbing in a quadruped robot. The development is based on the geometrical interactions between the robot legs and the stair, starting from single-leg analysis, followed by two-leg collaboration, and then four-leg coordination. In addition, a brief study on stability of the robot is also reported. Finally, simulation and experimental test are also executed to evaluate the performance of the algorithm.

## I. INTRODUCTION

Locomotion of the robots on the uneven terrains draw great attention in recent years. Lots of robots have been reported with great mobility on the nature rough terrains. However, a much less research is related to stair climbing. Shrimp rover [1] has clever mechanism design which combines wheels and self-adjustable linkages to maintain suitable body posture and to increase its mobility on uneven terrains and stairs. Loper [2] climbs stairs by rotating four Tri-lobe wheels. IMPASS [3] climbs obstacle driven by two rimless spoke wheels with two degree of freedoms. Some tracked robots [4] also utilize the treads on the tracks which on certain level can grab the edges of the stairs. The humanoid robot ASIMO [5] developed by HONDA demonstrates stair climbing behavior quite frequently in various robot shows. Hexapod ASTERISK [6] climbs the stair based on precise recognition of the stair by laser scanner. The hexapod robot RHex [7] also demonstrates excellent performance on stairs, including both stair ascent [8] and descent [9]. As for the quadrupeds, several of them are reported with great mobility; for example, Scout series [10], Tekken [11], Titan [12], and etc. RIMHO [13] climbs the stairs by utilizing various sensory feedbacks such as contact sensors, inclinometers, and joint sensors. Recently, the DARPA Learning Locomotion Program which uses BDI-developed small quadruped robot Little Dog as a common platform also includes stair climbing as one of the tasks. However, due to its small size comparing to the stair, the gait developed for this platform usually performs in an intermittent manner. In short, literature about stair climbing in quadrupeds is still very limited.

Along with our goal to drive *Quattroped* [14] agilely in

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various terrains, it motivates us to investigate the stair climbing behavior in a general mid-size quadruped (i.e. body length around 40-80cm), which probably is the minimum size of the robot capable of continuous stair climbing. Because rolling contacts induced by half-circle legs of *Quattroped* during locomotion like stair climbing is not general, in this paper we would like to focus on the “point contact” legs and to construct the basic framework about how to treat the stair climbing problem from kinematic and geometrical point of views. The algorithm developed requires various assumptions but not designed for a particular robot, so it might be suitable for a larger family of mid-size quadrupeds.

Section II introduces the terminologies and assumptions utilized in the algorithm, followed by trajectory planning detailed in Section III, including analysis of a single leg, and coordination of two and four legs. Section IV briefly investigates quasi-static stability of a robot during stair climbing, and Section V reports the simulation and experimental result. Section VI concludes the work.

## II. TERMINOLOGIES AND ASSUMPTIONS

The goal of this paper is to construct a framework of steady stair climbing gait in a quadruped robot, including 4-leg trajectory generation and coordination among them. The design is performed in the work space then transformed into joint space for robot locomotion. Assumptions and associated terminologies used in the algorithm are listed below and depicted in Figure 1:

1. Stair: Characteristic length  $CL$  and slope  $\phi$  of the stair are defined as  $CL = \sqrt{W^2 + H^2}$  and  $\phi = \arctan(H/W)$ , respectively, where  $W$  and  $H$  are width and height of each step. Empirically measured nominal value (and the standard deviation) of  $W$  and  $H$  of domestic stairs are 27cm (2.8cm) and 17cm (1.2cm), accordingly.

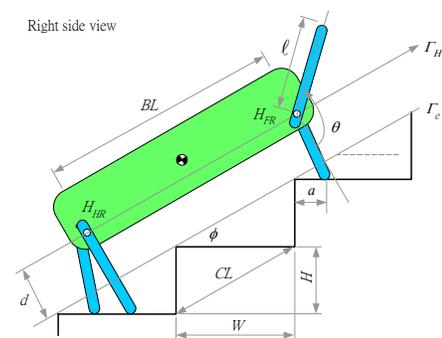


Fig. 1. Illustrative drawing of a quadruped which climbs the stair.

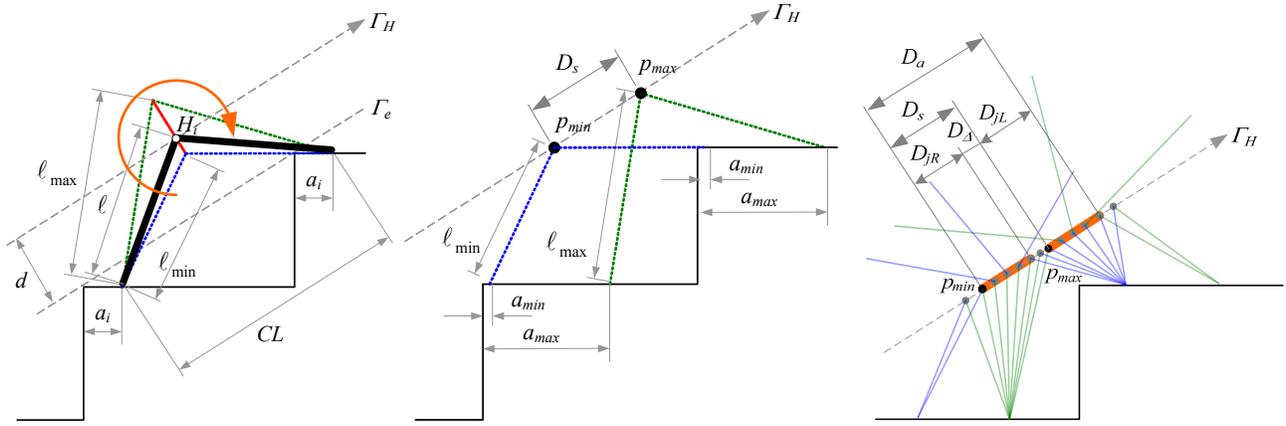


Fig. 2. Left: Configurations of a leg “right before” the lift-off of the foot from the lower step (left black bar) and “right after” the touchdown of that on the upper step (right black bar). Middle: Schematic diagram of two extremes: the earliest timing (blue line) and the latest (green line) timing to start swing the leg from the lower step to the upper one. Right: Illustrative drawing which shows swings of the left and right legs from the lower step to the upper one.

2. Individual leg motion: Each leg  $L_{i, i=FR,FL,HR,HL}$  is required to be capable of two degree-of-freedom (DOF) planar motion in the sagittal plane. The subscripts  $FR$ ,  $FL$ ,  $HR$ , and  $HL$  denote front right, front left, hind right, and hind left legs, respectively. For simplicity, foot position of each leg will be represented in the polar coordinate  $(l, \theta)$  with its origin located at the hip joint of each leg  $H_{i, i=FR,FL,HR,HL}$ . Please remind that 2 DOF articulated leg is also compatible with the following development since its coordinates  $(\theta, \theta_2)$  can be transformation into the polar one in straight-forward trigonometric operations.
3. Leg arrangement: Front/back hip joints of right and left legs ( $H_{FR}$  and  $H_{FL}$  /  $H_{HR}$  and  $H_{HL}$ ) are coincided at the same point from side view. Body length  $BL$  is defined as the distance between the front hip joint  $H_{FR}$  (or  $H_{FL}$ ) and the hind hip joint  $H_{HR}$  (or  $H_{HL}$ ).
4. Robot motion: The robot body is assumed to be operated in the quasi-static constant-velocity forward motion without any pitch and roll motion, and dynamics of the swing leg is ignored due to its low inertia compared to that of the body. In addition, foot is the only ground-contact portion of the robot during locomotion, like that of quadruped animals in general. These assumptions have the following implications: (1) The front and hind hip joints ( $H_{FR}/H_{FL}$  and  $H_{HR}/H_{HL}$ ) are moved along with the same trajectory  $\Gamma_H$ . It also has the same slope as the stair  $\phi$  and is located with an offset distance  $d$  from the line connected by edges of the steps  $\Gamma_e$  ( $d$  hereafter referred as hip clearance). (2) At least three feet touches the steps at every moment to maintain static stability (i.e. at most one leg is allowed to swing in the air at any time). (3) Geometrically, the stair is the periodic composition of steps with certain width and height. Thus, the nominal trajectory of each foot is scheduled to be moved in the periodic manner from one step to another step. For simplicity, the distance between the foot ground-contact point  $C_{i, i=FR,FL,RR,RL}$  and the edge of the step,  $a_{i, i=FR,FL,RR,RL}$ , is assumed fixed (i.e. “periodic 1” motion; a hereafter referred as contact offset).

### III. TRAJECTORY PLANNING

#### A. Trajectory analysis of a single leg

Figure 2(left) depicts the geometrical configurations of a leg  $L_i$  “right before” the lift-off of the foot from the lower step (left black bar) and “right after” the touchdown of that on the upper step (right black bar) with given arbitrary  $a_i$ . To simplify the design process, temporarily we assume that the leg lengths at the lift-off and the touchdown are equal, and the time duration for the leg to swing from lift-off to touchdown is infinitesimal. These assumptions add conservative constraints on the estimation of the required leg length, and those will be released in the following sections to meet real situations.

To avoid collision between non-foot portion of the leg and the edge of the step during leg swing from the lower step to the upper one, the length of the leg  $\ell$  needs to be long enough to lift the hip joint  $H_{i, i=FR,FL,HR,HL}$  above the horizontal surface of the upper step as shown in Figure 2(left). The right blue dashed line indicates the lowest configuration of the leg  $L_i$  to meet this requirement. Figure 2(left) clearly shows that  $\ell_{\min}$  is purely determined by the geometry of the stair,

$$\ell_{\min} = \frac{CL}{2} \sec \phi = \frac{W^2 + H^2}{2W} = \frac{1}{2}(W + H \tan \phi). \quad (1)$$

In the mean time, the distance  $a$  is constrained as well,

$$a \leq \ell_{\min}, \quad (2)$$

and this inequality avoids hitting of the hip joint to the stair. Please note that  $\ell_{\min}$  doesn't mean the minimum length the leg can achieve, but the minimum length the leg should have in order to perform a successful swing from the lower step to the upper one. With the nominal values  $W=27cm$  and  $H=17cm$ , the  $\ell_{\min}$  is calculated  $18.85cm$ . In addition, practically the robot also has its maximum operable leg length  $\ell_{\max}$  (green dashed line). Therefore, these two constraints bound the feasible leg length during stair climbing  $\ell_{\min} \leq \ell \leq \ell_{\max}$ , and these further confine the possible locations of the hip joint  $H_{i, i=FR,FL,HR,HL}$  on the red line shown

in Figure 2(left). It is also clearly shown in the figure that the longer red line indicates the wider range of  $d$  we can select.

Figure 2(left) also reveals that the contact offset  $a_i$  is closely related to the hip clearance  $d$ , the geometrical configuration of the steps ( $CL$  and  $\phi$ ), and the length of the robot leg  $\ell$  :

$$a_i = \left( \sqrt{\ell^2 - \frac{1}{4}CL^2} - d \right) \csc \phi \quad (3)$$

When the leg length equals to its minimum required value  $\ell_{\min}$ , the equation can be simplified to:

$$a_i = \ell_{\min} - d \csc \phi, \quad (4)$$

where  $a_i$  reaches its maximum equal to  $\ell_{\min}$  shown in (2) when  $d$  approaches 0. In general, hip clearance  $d$  is an active variable for which we have a preferred value. Thus, combining  $d$  with possible range of leg length  $\ell_{\min} \leq \ell \leq \ell_{\max}$ , the feasible range of contact offset  $a_i$  can be derived by (3). It shows that the value  $a_i$  can be varied with given maximum leg length  $\ell_{\max}$  and the hip clearance  $d$ . It reveals two trends: First, if robot's leg has a larger  $\ell_{\max}$ , a wider range of  $a_i$  is possible. Second, to have a larger hip clearance  $d$  implies to use a smaller contact offset  $a_i$ , which means the foot should contact with the step close to the step edge. In addition, the feasible range of  $a_i$  also determines how different the timings for right and left legs to swing from the lower step to the upper one, which further decides whether the coordination among legs is feasible or not. The details of the related materials will be illustrated in the following section.

### B. Coordination between two front or hind legs

If only a single leg with the assumptions defined in the Section II is considered, the leg is capable of moving from one step to another step as long as the length of the leg is longer than the minimum required length  $\ell_{\min}$ . However, this requirement is not sufficient if the motions of two front or hind legs are considered together due to the fact that two legs cannot swing simultaneously in order to maintain static stability of the robot (i.e. at least three legs on the ground). Since hip joints of the right and left legs are coincides to the same point ( $H_{FR}$  and  $H_{FL}$  /  $H_{HR}$  and  $H_{HL}$ ) from side view, different timings for right and left legs to swing also mean different geometrical locations of the hip joints  $H_i$  with respect to the stair when the right or left legs swings. Thus, with given maximum leg length  $\ell_{\max}$  and hip clearance  $d$ , the feasible positions of  $H_i$  for leg swing is shown in Figure 2(middle). The earliest timing is when the hip joint  $H_i$  reaches point  $p_{\min}$  (at which the leg length is equal to  $\ell_{\min}$ ), and the latest timing is when that reaches  $p_{\max}$  (at which the leg length is equal to  $\ell_{\max}$ ). Please note that the contact offset  $a_i$  is also at its two extremes  $a_{\min}$  and  $a_{\max}$  with hip joints arriving at  $p_{\min}$  and  $p_{\max}$ . This figure also clearly reveals that longer  $\ell_{\max}$  allows bigger difference of the configurations (or timings) for the right and left legs to swing. In practical the

motor has a torque limit, thus there must exist certain difference between  $p_{\min}$  and  $p_{\max}$ , so the swing motion can be achieved in reality. Though wider separation of  $p_{\min}$  and  $p_{\max}$  can reduce the requirement of the motor torque, it indeed has an upper limit due to possible interference of coordination among all 4 legs, and this effect will be described in details in the following section.

In short, the analysis so far can be summarized into a standard design procedure described below:

1. Assign the maximum leg length  $\ell_{\max}$ .
2. Define the desired hip clearance  $d$ .
3. Derive the contact offset  $a$  of the right leg,  $a_R$ , from (4). Here we assume the right leg swings at its earliest possible timing, where the leg length is equal to  $\ell_{\min}$  and  $a_R$  equals to  $a_{\min}$ .
4. Derive the contact offset  $a$  of the left leg,  $a_L$ , from (3) with  $\ell$  equals to  $\ell_{\max}$ . Here we assume the left leg swings at its latest possible timing, where the leg length is equal to  $\ell_{\max}$  and  $a_L$  equals to  $a_{\max}$ .

As mentioned before, the timing to swing the leg is strongly determined by the position of the hip joint  $H_i$  relative to the step. Once the hip joint reaches certain position along the motion trajectory  $\Gamma_H$ , the associate leg  $L_i$  is required to initiate the swing motion as depicted in Figure 2(middle). If the leg starts to swing early than it should, it may hit the edge of the stair (for the blue leg only) or not be able to reach the upper stair at designate location with contact offset  $a_i$  due to length constraint. In contrary, if the leg doesn't start to swing at the location it should, it may not be able to reach the new contact at the upper stair in time. Both cases will cause instability to the robot locomotion. Thus, how to arrange adequate timings to swing all four legs in sequence within one characteristic length  $CL$  is the essential task for stable robot locomotion.

In reality, the swing of a leg from lift-off position at the lower step to touchdown position at the upper step takes certain amount of time due to limitation of the motor torque. Thus, the positions of the hip joint  $H_{i, i=FR,FL,HR,HL}$  at lift-off and at touchdown will be different on the trajectory  $\Gamma_H$  based on the assumption of constant-velocity robot locomotion described in the Section II. Figure 2(right) depicts several snap shots of leg motions during swings, and the green region illustrates the traveling distance  $D_{i, i=FR,FL,HR,HL}$  of hip  $H_i$  for the leg  $L_i$  to complete a swing. It is obvious that the timing to lift-off the left leg (blue color) cannot happen before the touchdown of the right leg (green color) in order to maintain static stability. On the safe side, a buffer time  $D_A$  is also designed to avoid any mis-coordination due to any motion delay.

Following the assumptions that the right leg starts to swing at its earliest time ( $\ell = \ell_{\min}$ , blue color) and the left leg swings at its latest time ( $\ell = \ell_{\max}$ , green color) shown in Figure 2(middle), the position (or time) difference  $D_s$  at

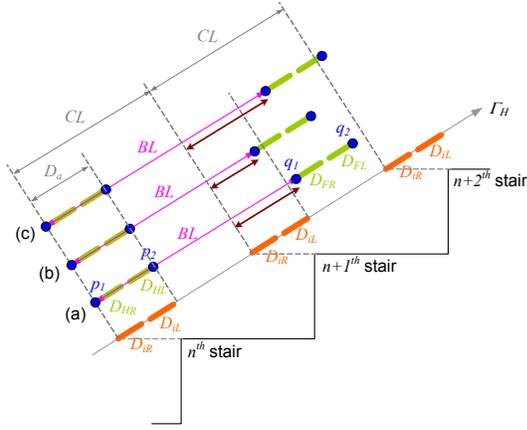


Fig. 3. Three typical scenarios of 4-leg coordination: (a) four legs can swings from the lower step to the upper step periodically and sequentially without any problem. (b) interference --- the hind right leg needs to start swing while the front left leg is still in the swing phase. (c) interference --- the front right leg needs to start swing while the hind left leg is still in the swing phase. Brown lines indicates  $rem(BL, CL)$  used in (8) and (9).

which the right and left leg start to swing can be calculated quantitatively:

$$D_s = \frac{W}{H} \sqrt{\ell_{\max}^2 - \frac{1}{4}(W^2 + H^2)} - \frac{1}{2} \sqrt{W^2 + H^2}. \quad (5)$$

Assuming all legs have equal  $D_i$  and  $p$  percent of time is reserved as the buffer period  $D_A$  (i.e.  $p_A = D_A/D_s$ ) between the right and left  $D_s$  ( $D_{jR}$  and  $D_{jL}, j=F,H$ ) shown in Figure 2(right), the duration for each leg to swing  $D_i$  can be computed

$$D_i = (1-p) D_s = (1-p) \left[ \frac{W}{H} \sqrt{\ell_{\max}^2 - \frac{1}{4}(W^2 + H^2)} - \frac{1}{2} \sqrt{W^2 + H^2} \right], \quad (6)$$

and the complete duration  $D_a$  of left and right leg pair can be derived as

$$D_a = (2-p) D_s = (2-p) \left[ \frac{W}{H} \sqrt{\ell_{\max}^2 - \frac{1}{4}(W^2 + H^2)} - \frac{1}{2} \sqrt{W^2 + H^2} \right]. \quad (7)$$

Please note that the swings of the right and left legs of front or hind leg pairs ( $D_{FR}$  and  $D_{FL}$  or  $D_{HR}$  and  $D_{HL}$ ) in this paper is set to be executed contiguously. It is not feasible to swing the legs in the sequence of front-hind-front-hind legs because of possible collision to the edge of the step and of leg length constraint  $\ell_{\max}$  as depicted in Figure 2(right). If  $\ell_{\max}$  is much larger than  $\ell_{\min}$  and if  $d$  is far from the stair, this constraint can be released. In this case the robot size is generally much larger comparing to that of the steps.

### C. Coordination among all four legs

Stair climbing from step to step is a periodic motion, so it is intuitive that all four legs are required to complete the swings while the hip joints or the robot body travels within one  $CL$ . In addition, since right and left legs should swing consecutively, it is more convenient to analysis 4-leg coordination by considering arrangement of two  $D_a$  s in one  $CL$ , one for front legs and one for hind legs. Thus, it is intuitive that  $D_a$  should be no more than half of  $CL$ , or it is not possible to put two  $D_a$  s, in one  $CL$ , and this further means swing of all four legs can't be done in one step of stair climbing.

The adequate arrangement of two  $D_a$  s in one  $CL$  depends

on two parameters: one is the duration of  $D_a$ , and the other is the body length  $BL$ . Starting point  $p_{\min}$  of  $D_a$  is located at a specific location with respect to the step, so the freedom to tune the duration of  $D_a$  lies in the selection of  $\ell_{\max}$

determines the end point  $p_{\max}$ . After a specific  $D_a$  is selected, whether two  $D_a$  s are overlapped with each other or not is strongly determined by  $BL$ . Figure 3 shows three different scenarios: in (a) four legs can swing periodically and sequentially without any problem, but in (b) and (c) there exists certain interference between front leg and hind legs. In Figure 3 positions of the hind hip joints  $H_{HR}/H_{HL}$  in all three scenarios are all aligned with the position  $p_1$ , where the hind legs are required to start the swing phase. In the meantime, the front hip joints  $H_{FR}/H_{FL}$  are located at  $q_1$ . The swing phase of the hind legs ends when the hind hip joints  $H_{HR}/H_{HL}$  arrive at  $p_2$ , while the front ones arrive at  $q_2$ . Therefore, it is obvious that in (a) during the whole swing phase  $p_1$  to  $p_2$  of the hind legs, the front hip moves from  $q_1$  to  $q_2$  and it is not in the swing phase; thus the adequate swings of all four legs is possible. However, (b) indicates that the hind right leg needs to start swing (arriving  $p_1$ ) while the front left leg is still in swing phase, and (c) indicates that the front right leg needs to start swing while the hind left leg does not finish yet. Both (b) and (c) have certain time period where two legs are needed in the swing phase; thus the assumption of static stability can not be maintained thoroughly in one  $CL$  motion cycle.

The quantitative criteria to avoid interference among 4 legs like (b) and (c) shown in Figure 3 can be summarized as follows:

$$rem(BL, CL) + D_a \leq CL \quad (8)$$

$$rem(BL, CL) \geq D_a, \quad (9)$$

where  $rem(x,y)$  represents the function to acquire remainder of  $x/y$ , and its length is plotted in brown color in Figure 3. In addition, the constraint which grants the nonzero duration  $D_a$ :

$$\ell_{\max} > \ell_{\min}. \quad (10)$$

These inequalities represent important relations among dimensions of the stairs,  $W$  and  $H$ , and lengths of the robot body  $BL$  and legs  $\ell$ . Assuming the body length is designed within the range  $CL < BL < 2CL$ , equations (8)-(10) can further be simplified to:

$$(BL - CL) + D_a(W, H, \ell_{\max}, p) \leq CL \quad (11)$$

$$(BL - CL) \geq D_a(W, H, \ell_{\max}, p) \quad (12)$$

$$\ell_{\max} > (W^2 + H^2) / 2W. \quad (13)$$

These inequalities are constructed by four major independent variables ( $W$ ,  $H$ ,  $\ell_{\max}$ , and  $BL$ ) and one minor variable  $p$ .

Dimensionless inequalities are computed by dividing the first three variables by the last one  $BL$  shown below:

$$\left(\frac{\ell_{\max}}{BL}\right)^2 \leq \left(\frac{1}{2} \frac{CL}{BL}\right)^2 + \left[ \left(3 - \frac{p}{2}\right) \frac{CL}{BL} - 1 \right]^2 \left[ (2-p) \left(\frac{W}{BL}\right) \left(\frac{H}{BL}\right) \right]^{-2} \quad (14)$$

$$\left(\frac{\ell_{\max}}{BL}\right)^2 \leq \left(\frac{1}{2} \frac{CL}{BL}\right)^2 + \left[ \frac{p}{2} \frac{CL}{BL} - 1 \right]^2 \left[ (2-p) \left(\frac{W}{BL}\right) \left(\frac{H}{BL}\right) \right]^{-2} \quad (15)$$

$$\left(\frac{\ell_{\max}}{BL}\right) \geq \frac{1}{2} \left(\frac{CL}{BL}\right)^2 \left(\frac{W}{BL}\right) \quad (16)$$

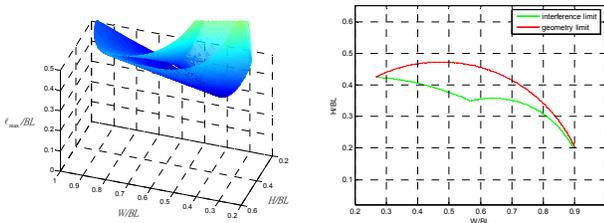


Fig. 4. Left: Area bounded by three surfaces indicates the feasible selection of dimensions which satisfies the inequality constraints (14)-(16). Right: The feasible relative dimensions of  $W$  and  $H$  to  $BL$  is bounded by three curves derived in (14)-(16).

Both interference scenarios indicate that  $\ell_{\max}/BL$  is smaller than a specific value with given  $W$ ,  $H$ ,  $BL$ , and  $p$ . This means interference only limits upper bound of  $\ell_{\max}$ . As for “which case” bounds extreme value of  $\ell_{\max}$  is depending on the given values of  $W$ ,  $H$ ,  $BL$ , and  $p$ . In general, values of  $H$  and  $\ell_{\max}$  are usually smaller than one half of  $BL$ .

The inequalities shown in (14)-(16) is plotted in Figure 4(left), where the coordinates of three axes are dimensionless  $W/BL$ ,  $H/BL$ , and  $\ell_{\max}/BL$ . The volume enclosed by these three surfaces indicates the feasible relations among these variables for successful 4-leg coordination. With the selections of  $\ell_{\max}=21\text{cm}$ ,  $BL=44.4\text{cm}$ , and  $p=42\%$ , the 2D cross section can be extracted from the 3D plot as shown in Figure 4(right), which is very informative in selecting suitable variables, such as  $BL$  with given  $W$  and  $H$ . Please remind that from (8)/(10) to (11)/(12) is based on the assumption of  $CL < BL < 2CL$ . For different relations will yield different (11)/(12), which further derive different (14)/(15). In the current selections of parameters  $D_a$  is around 25.4%, which leaves certain room for tuning  $BL$  with respect to give  $W$  and  $H$ . If  $D_a$  reaches its maximum (i.e. half of the  $CL$ ), the swing of the front and hind leg pairs happens consecutively, and the body length is strictly confined to  $BL(n - 0.5)$ , where  $n$  is a positive integer.

#### IV. STABILITY ANALYSIS

Analysis above is focused on the motion planning on the sagittal plane. In reality the roll balance is also needed to be

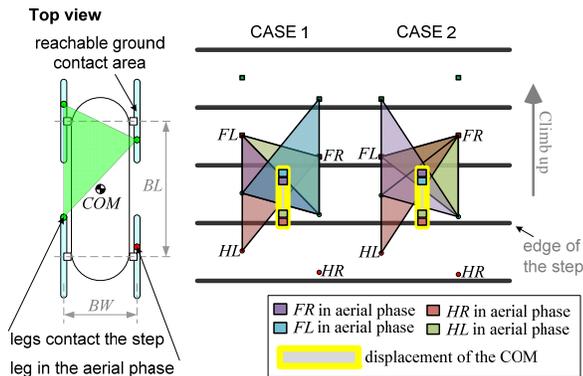


Fig. 5. Quasi-static stability analysis of the robot: the geometrical relation between COM of the robot and the triangle formed by three stair-contact legs.

considered, especially when the robot contacts the stair by three legs only.

Figure 5(left) shows that there exists certain moments that the COM might fall out of the instantaneous triangle region formed by three ground contact points, so the contact points of the legs on the stairs should be chosen carefully (i.e. chosen of the contact offset  $a_i$ ). Two possible arrangements of  $a_i$  are found based on permutation of the legs shown in Figure 5(right). In the first case both right legs contact the step with  $a_{\min}$ , where the leg lengths are equal to  $\ell_{\min}$  shown in Figure 2(middle), and both left legs contact the step with  $a_{\max}$ , where the leg lengths are equal to  $\ell_{\max}$ . This is the scenario used in the precious sections. In this case, the swing sequences of all four legs in one period is  $L_{HR}$ ,  $L_{HL}$ ,  $L_{FR}$ , and  $L_{FL}$ . In the other case  $a_i$ s are switched in the front leg pairs. Thus, the swing sequences of all four legs in one period is  $L_{HR}$ ,  $L_{HL}$ ,  $L_{FL}$ , and  $L_{FR}$ . The quasi-static stability of the robot during stair climbing can be analyzed by relative location of the COM with respect to the contact triangles formed by three step-contact legs shown in Figure 5. In both cases each triangle indicates the instant where one specific leg is in swing phase, and the square mark in the same color as the triangle shows position of the COM in that instant. This figure clearly shows that the robot would keep balanced as the front legs swing from the lower step to the upper one, but it may lose balance as the hind legs swing since the COM falls out of the contact triangle. By using the same set of parameters  $BL=44.4\text{cm}$ ,  $BW=36\text{cm}$ ,  $\ell_{\max}=21\text{cm}$ ,  $p=42\%$ , percentages of time where the COM of the robot falls out of the contact triangle in case 1 and 2 are 27.3% and 30%, respectively. The actual pitching and rolling behaviors of the robot due to this unbalanced moment is further determined by its time duration and the dynamics of the robots. Further discussion will be described in the experimental section.

#### V. SIMULATION AND EXPERIMENTAL RESULTS

The algorithm developed in the previous sections has been simulated in Matlab with a particular set of parameters ( $W=27$ ,  $H=17$ ,  $d=8.5$ ,  $L=21$ ,  $BL=44.4$ , and  $p=41\%$ ) which matches the parameters of *Quattroped* [14] and of general local stairs. Figure 6 shows sequential snapshots of simulated result within climbing of one step. The full simulation is available as the supplemental material associated with this paper. The simulation shows that the legs can be coordinated to swing from the lower step to the upper one in sequences within one travel distance  $CL$  of COM without any interference, as expected from the analysis.

The algorithm was implemented in the robot *Quattroped* and evaluated experimentally. Figure 7 shows the sequential images extracted from the video recording of robot climbing. The robot configuration of 14 snapshots also corresponds to those of 14 simulation subplots shown in Figure 6. The full video is also available as the supplemental material associated with this paper. The video confirms that the algorithm is

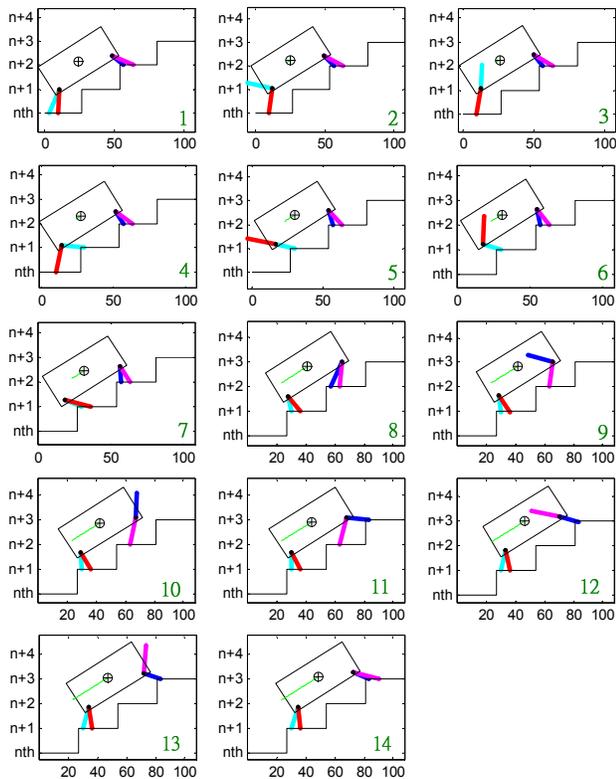


Fig. 6. Sequential snapshots of the robot climbing the stair in simulation environment. The robot body and its COM are plotted as a rectangle and a cross within a circle. The magenta, blue, cyan, and red colors indicate front right, front left, hind right, and hind left legs, respectively. Unit of horizontal axis: cm, unit of vertical axis: number of step.

functional and the robot can indeed climb the stair. Empirically the robot will indeed have pitching and rolling behaviors during the moments where the COM falls out of the contact triangles as described in the previous section, but these behaviors are corrected adequately after the swing leg contacts the step. The body is free of contact the steps in the whole process.

## VI. CONCLUSION AND FUTURE WORKS

We report on the algorithm of trajectory planning and four leg coordination for quasi-static stair climbing in a quadruped robot. The detailed development is based on the geometrical interactions between the robot legs and the stair. The suitable characteristic dimensions of the robot and how these parameters affect the algorithm are demonstrated. In addition, a brief study on the quasi-static stability of the robot shows that the stability can be maintained most of time and the possible unstable postures can be corrected by the followed stable four-leg supporting posture. Finally, the algorithm is simulated and evaluated experimentally, which confirms the proposed algorithm is functional.

We are currently in the process of developing feedback mechanism of the algorithm, which will further tolerate a much wider geometrical variations of the stair. In the meantime, the dynamics of the system is under investigation as well.

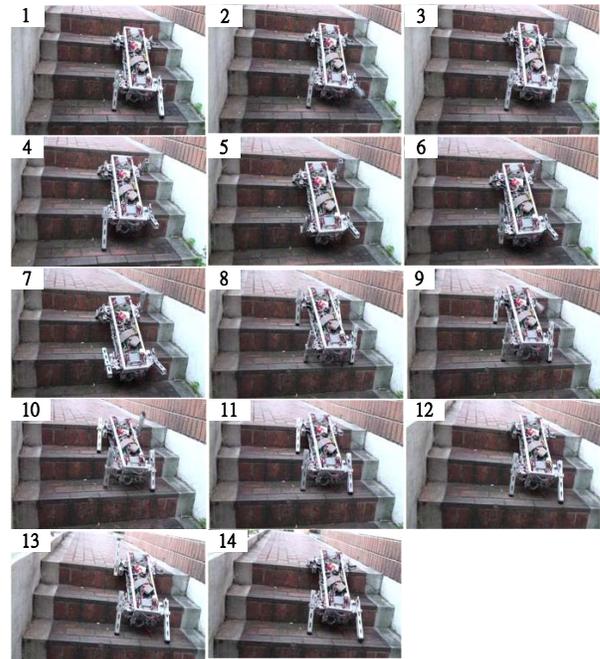


Fig. 7. Sequential images of the robot *Quattroped* climbing the stair

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