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Trajectory Planning for Stair Climbing in the Leg-wheel Hybrid Mobile Robot *Quattroped*

Shen-Chiang Chen, Ke Jung Huang, Cheng-Hsin Li, and Pei-Chun Lin

Abstract—**The algorithm of trajectory planning and four leg coordination for quasi-static stair climbing in a quadruped robot is reported. The development is based on the geometrical interactions between the robot legs and the stair, starting from single-leg analysis, followed by two-leg collaboration, and then four-leg coordination. A brief study on stability of the robot is included. In addition, the first-step and last-step algorithm which deals with the transient between the flat ground and the stair is reported as well. Finally, simulation and experimental validation are performed to evaluate the performance of the algorithm.**

I. Introduction

Legs and wheels are two widely adopted methodologies utilized in ground locomotion platforms. After a long evolutionary process, the legs of most of ground animals are agile and robust and are capable of driving animals to move smoothly and rapidly on uneven natural terrains. Wheels, on the contrary, were invented by humans for specialized locomotion on flat ground; their excellent performance in power efficiency and smooth travel with high speed on flat ground sets a high standard with which legged locomotion can hardly compete. Thus, during the concept design process, we aimed to design a leg-wheel hybrid platform with great mobility on both flat ground (by wheels) and rough terrain (by legs), to provide an adequate combination of mobile platforms suitable for general indoor-outdoor and flat-rough environments.

Lots of legged or hybrid robots have been reported with great mobility on the uneven terrains. Comparing to that, however, only a few research deals with stair climbing. Shrimp rover [1] has special mechanism design which combines wheels and self-adjustable linkages to maintain suitable body posture and to increase its mobility on uneven terrains and stairs. Loper [2] climbs stairs by rotating four Tri-lobe wheels. IMPASS [3, 4] climbs obstacle driven by two rimless spoke wheels with two degree of freedoms. Some legged-tracked hybrid robots [5, 6] utilize the treads on the tracks which on certain level can grab the edges of the stairs and thus climbs on them. The humanoid robot ASIMO developed by HONDA demonstrates stair climbing behavior quite frequently in various robot shows. Hexapod ASTERISK [7] climbs the stair based on precise recognition of the stair by laser scanner. The hexapod robot RHex [8] also demonstrates excellent performance on stairs, including both stair ascent [9, 10], and stair descent [11]. The quadruped robot Scout [12] can climb the step by careful maneuver of body dynamics. RIMHO [13] climbs the stairs quasi-statically by utilizing various sensory feedbacks such as contact sensors, inclinometers, and joint sensors. Recently, the DARPA Learning Locomotion Program which uses BDI-developed small quadruped robot Little Dog as a common platform also includes stair climbing as one of the tasks.

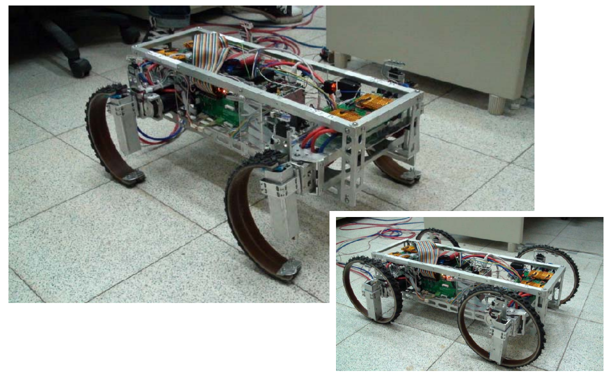

Fig. 1. Picture of the *Quattroped*, a leg-wheel hybrid mobile robot.

Previously, we had reported on the stair climbing of a quadruped with point-foot legs [14]. During stair climbing contact point of each leg on each step is assumed fixed as most legged robots do. Here, we investigate feasibility of stair climbing in the 4-leg/4-wheel hybrid mobile platform *Quattroped* shown in Fig. 1. Comparing to most hybrid platforms which have separate mechanisms of wheels and legs, this robot is implemented with a transformation mechanism which directly changes the morphology of wheels (i.e. a full circle) into half-circle legs, each with 2 active degree-of-freedom (DOF) [15]. Thus, during locomotion the leg/wheel may performs fixed point contact or rolling behaviors depended on the configuration of the robot with respect to the stairs. Thus, a more generalized methodology is developed and reported in this paper, which allows movements of the contact points while the robot moves. The developed algorithm requires various assumptions but not designed for any particular robot, so it might be suitable for a larger family of mid-size robots with 40-80cm body length.

Section II introduces essential information regarding the robot and the stairs, followed by trajectory planning of stair climbing detailed in section III. Section IV briefly describes the stability consideration, and Section V introduces the algorithms of the ground-stair transient in the first and last steps. Section VI reports the simulation and experimental

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Authors are with Department of Mechanical Engineering, National Taiwan University (NTU), No.1 Roosevelt Rd. Sec.4, Taipei, Taiwan. (Corresponding email: peichunlin@ntu.edu.tw).

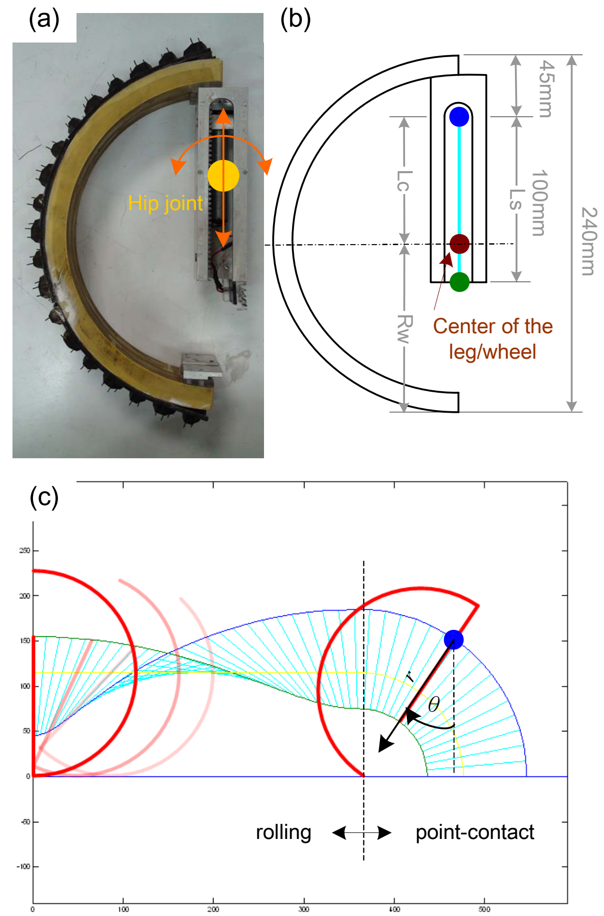

Fig. 2. (a)(b) Picture and dimensions of the half-circle leg of the robot *Quattroped*; (c) Motion of the leg on the smooth ground.

results, and Section VII concludes the work.

II. The Platform and the Stairs

The photo of *Quattroped* is shown in Fig. 1. This platform has a "transformation mechanism" capable of deforming a specific portion of the body to act as a wheel or as a leg. It uses the wheel mode to drive on smooth grounds, and it switches to the leg mode to pass rough terrains by folding the rims of the wheels into the half-circle legs. The robot weighs 14.5kg and has dimensions of 600mm in length, 300mm in width, and 260mm in height (in legged standing mode), respectively. The distance between hips of rear legs/wheels and those of front ones is 450mm, and the distance of the hips to the bottom of the robot is 65mm. The robot has totally 8 actuated DOFs on four legs/wheels for driving, and it also has two extra DOFs on the front leg/wheels for turning while operating in the wheeled mode.

Side view of the half-circle leg (i.e., leg/wheel in the legged mode) is shown in Fig. 2(a). Basically, the leg has 2 active DOFs performing in the polar coordinate (r, θ). More specifically, the leg can rotate and move linearly with respect to the hip, which is defined as the connecting point from the body to the leg/wheel and is fixed on the body. As shown in Fig. 2(b), the radius of the leg/wheel (Rw) is 120mm. The workspace of the linear motion (Ls) is 100mm, where both ends of this DOF (marked in a blue dot and a green dot, respectively) are 75cm and 25cm apart the center of the leg/wheel (marked in a brown dot).

The general forward motion of the half-circle leg moving on the ground consists of two states shown in Fig. 2(c). The leg rolls on the ground while the ground-contact point is on the surface of the whole half-circle, and after that the leg moves with a single fixed point contacting the ground. Following the same color notations as in the Fig. 2(b), at every instant the possible positions of the hips are marked between green and blue dots, and some of the feasible regions are plotted in cyan lines for the demonstration purpose. Thus, the complete workspace of the hip during the leg motion is bounded by blue and green lines shown in Fig. 2(c). Please note that in the analysis of general manipulators, the base is fixed and the workspace of the end-effector is plotted. In the contrary, the configuration of the body (or hip) in the mobile platforms is reversely determined by that of the leg/wheel related to the ground. The so-called "base" is not a fixed point anymore and is passively determined by how the mobile platforms interact with the contact surfaces. In this case, how the leg/wheel moves on the ground becomes an active factor. Thus, though the workspace of each leg with respect to the "fixed" body can be plotted, the process should be reversed by plotting the workspace of the hip given certain leg/wheel motion on the ground.

In general, the stair can be treated as the raise of the surface which is paralleled to the current one with a specific height and with a specific depth from the edge of the current raised surface (hereafter referred as a "step" of the stair). Thus, the geometrical description of the stairs can usually be quantified by the terminologies of the depth (W) and the height (H) of each step of the stairs. Practical measurements of the domestic stairs indicate that the common range of W is between 250 to 300mm and that of H is between 150 to 200mm.

III. Trajectory Planning

The leg trajectories of the robot should be operated in a periodic manner on each step during stair climbing since the geometry of the stair is composed by the steps periodically with similar dimensions. The depth of the step actively defines the feasible region for the leg to operate in the rolling phase, and the height of the step determines whether the workspaces of the leg on two consecutive steps have overlapped region or not. With the dimensions of W=300 mm and H=180 mm, Fig. 3(a) shows that two workspaces of the legs between consecutive steps have no overlapped region if the robot climbs the stairs in the designated forward direction. Thus, the trajectory of the hip cannot be generated from the lower step to the upper step in a continuous manner, unless the hip can keep moving up and enter the upper workspace successfully after the contact to the lower step loses (i.e., leave the lower workspace). In this scenario the robot actually moves in the so-called aerial-phase locomotion, which requires the generation of appropriated initial velocity while leaving the current workspace (i.e., the take-off motion). This

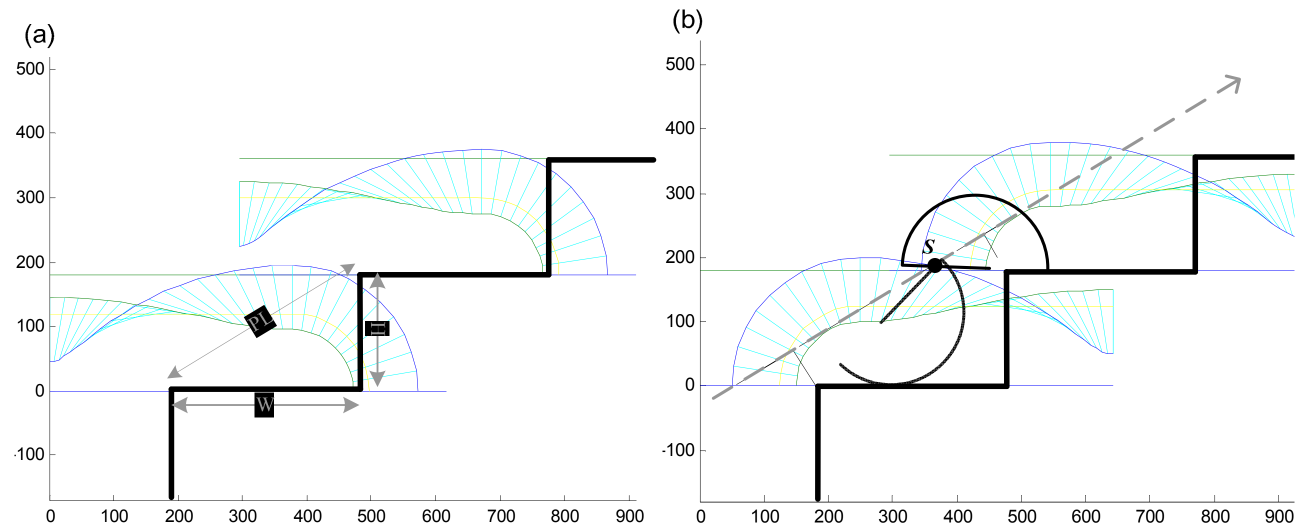

Fig. 3. The workspaces of the legs: (a) the robot moves in the forward direction; (b) the robot moves backward.

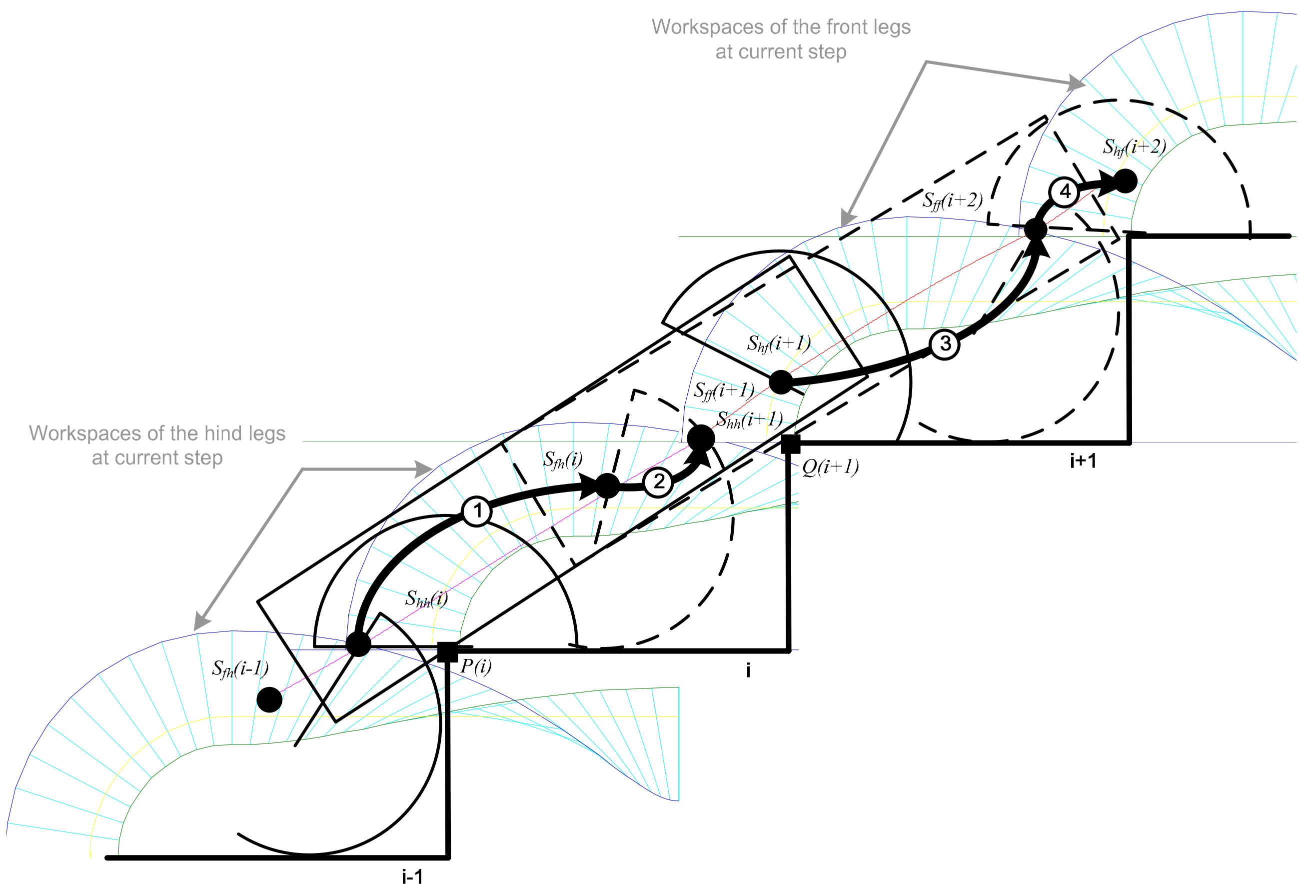

Fig. 4. Illustrative plot of the robot climbing the stairs.

kind of behaviors can be observed quite often in the quadruped animals; however, it is rarely shown in the general walking quadruped robots due to the limitation of the joint power density. Therefore, in the following development the robot is operated in the backward direction, where two workspaces of the legs between consecutive steps have an overlapped region as shown in Fig. 3(b). Fig. 3(b) also clearly shows that the right configuration for the leg to swing from the lower step to the upper step is while the hip arrives the point *S* (hereafter referred as the “characteristic point”). This is true for all four legs.

The next procedure lies in how to arrange the appropriate timings to swing all four legs while maintaining the stability and motion of the robot. In our previous work the quadruped with point-foot legs climbs the stairs with constant forward velocity, and the four legs swing one after another according to a designated sequence [14]. In this scenario, since the front or hind legs share the same hip position in the sagittal plane, different leg-swing timings also mean different hip positions relative to the steps. Thus, this can be achieved only if the overlapped region is large enough to be divided into two portions; each is allocated for one leg to swing. However, the same strategy is not functional with the half-circle configuration of the legs due to the much smaller overlapped region comparing to the point-foot one. Thus, a new strategy is developed--- while the front or hind hip arrive the point S shown in Fig. 3(b), the robot stops shortly and swings the front or hind legs to the upper step, followed by resuming its forward motion. This setup also indicates the robot is driven by all four legs during the climbing period, and the robot stops twice for leg swings while climbing each step with forward distance PL shown in Fig. 3(a).

After that, the task focuses on how to plan detailed trajectory of the front or hind hips from swing point $S_{ii}(i)$ $_{i=ff,hh}$ to the next swing point $S_{ii}(i+1)$, where f and h represent front and hind, respectively. Because the distance between the front and hind hips is fixed, the trajectories of the front and hinds ones cannot be planned separately. Thus, the trajectory of the hind hip is planned first, and that of the front one is generated based on this trajectory together with the rigid body geometry and assigned body pitch versus time. The details are described as follows. First, assume the trajectory of the hind hip is planned to move from point $S_{hh}(i)$ to point $S_{hh}(i+1)$ shown in Fig. 4. Both points represent the moments the hind legs swing, and at these moments the front legs are oriented in a special way such that the body touches the edge of the step $P(i)$. Before the hind hip reaches $S_{hh}(i+1)$, the front hip moves from $S_{hf}(i+1)$ and reaches point $S_{ff}(i+2)$ first, and at this instant the feasible positions of the hind hip are constrained in a small arc line due to the fixed distance between the hips. Along the arc, a point $Q(i+1)$ is selected, where the robot body lays on the edge of the step in order to improve the stability of the robot while the front legs swing. Therefore, the trajectory planning of the hind hip is divided into two segments: from $S_{hh}(i)$ to $S_{fh}(i)$ (route 1), and from $S_{fh}(i)$ to $S_{hh}(i+1)$ (route 2). The trajectories in the horizontal and vertical directions are also planned separately. For each segment, a fifth-order polynomial is utilized because of given six boundary conditions, including initial and final positions, velocities, and accelerations.

With given trajectory of the hind hip, similarly, the possible positions of the front hip at every instant reside in a small arc line due to the constraint of the fixed distance between the hips. In order to keep the continuity of the climbing motion as well as to avoid collision of the robot to the edge of the steps, trajectories of the body pitch while the hind hip moves from $S_{hh}(i)$ to $S_{fh}(i)$, and from $S_{fh}(i)$ to $S_{hh}(i+1)$ are also planned. Likewise, a fifth-order polynomial is utilized. Combining this two functions with route 1 and 2, the motion of the front hip from $S_{hf}(i+1)$ to $S_{ff}(i+2)$ (route 3), and from $S_{ff}(i+2)$ to $S_{hf}(i+2)$ (route 4) can be planned accordingly. The combined trajectory represents the complete trajectory of the front hip to climb one step of the stairs. Route 1 and 3 describe the trajectories of the hind and front hips right after the swing of the hind legs from step $i-1$ to step i, and route 2 and 4 describe the trajectories of the hind and front hips right after the swing of the front legs from step $i+1$ to step $i+2$.

Finally, with designed trajectories of the front and the hind hips, the followed procedure is to determine the joint trajectories $(r(t), \theta(t))$ of all four legs. The process is similar to the usage of inverse kinematics to find the joint angles in the standard robot manipulators. Here the hip trajectories are in some level equal to the trajectory of the "end-effector". As shown in Fig. 2(c), since the leg motion is hybrid, the computation of the joint trajectories should be processed separately. For the first portion while the leg contacts the ground at a point, the joint trajectories can be computed by geometric configuration directly. For the second portion while the leg rolls on the ground, the relation between the hip positions and the joint angle θ is nonlinear due to the cycloid rolling motion:

$$\frac{H_y - R_w}{H_x - O_x(\theta)} = \tan(\theta),$$

where H_x and H_y are the coordinates of the hips. O_x is the x-coordinate of the leg center, and it is linear proportional to θ. Because of the trigonometric function appeared in the equation, the θ is solved numerically by the root-finding algorithm.

IV. Stability Consideration

The locomotion stability of quadruped robots is one of the important issues needed to be considered while planning the trajectories. While the quadruped animals perform dynamic balance in their gaits such as walking, trotting, and galloping, the stair climbing of the robots is in some level confined in a quasi-static motion due to limitation of joint power density. Thus, in this case the stability property can be judged by checking the relative position of the COM to the supporting polygon. Unlike the tripod-gait hexapod robots whose COMs can be easily set around the centers of the supporting triangles, the lift-off of any leg in a quadruped can easily let the COM fall out of the supporting triangle, which is determined by the rest three ground-contact legs. In our previous work, the point-foot legs of the quadruped swing one after another according to a designated sequence [14], and we did observe the unwanted pitch and roll body motion while the leg swings. This is partially due to the relative positions of the COM with respect to the supporting triangle shown in Fig. 5 and partially due to the dynamics caused by leg swing motion. Therefore, the gait planned in this work is set in a more stable manner. First, four legs push the body cooperatively during climbing. Thus, even certain dynamics is generated to accelerate and decelerate the body; the COM (or ZMP) lies firmly in the supporting rectangle. Second, during the leg swing, the robot body is oriented to contact the edge of the step lightly, to provide two extra contact points as shown in Fig. 5.

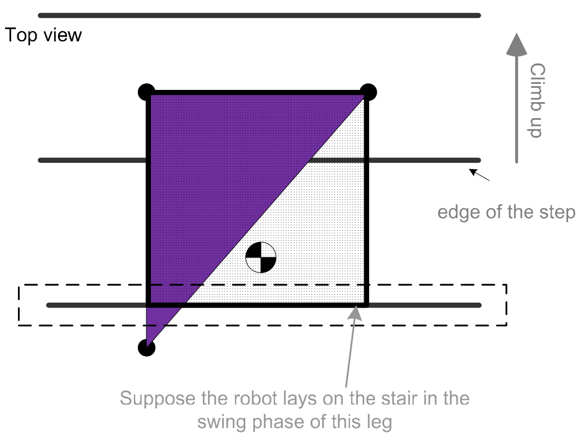

Fig. 5. Two different supporting conditions while the right hind leg is in swing phase---blue triangle: the robot is supported by three legs; red rectangular: the robot is supported by two front legs and the body with

Practically, not all three legs and these two points contact the ground simultaneously because of the dimension variation of the stairs. In some scenarios the robot is supported by two legs and two body-contact points, which form a stable supporting rectangle. In some other occasions the robot is still supported by three legs and the body is very close to the edge of the step; thus the range of pitch and roll motions can be confined to small values. In addition, due to the same practical concern, the front or hind two legs don't swing at the same time, but one after the other, to grant the life-off of the leg can be executed smoothly (i.e., not much force loaded on that leg right before lift-off).

V. First Step and Last Step Algorithm

The algorithm shown in the previous sections represents the steady-state motion which assumes the robot is well positioned on the stairs. The transition from the flat ground to the first-step of the stair and that from the last-step of the stair to the flat ground requires further development detailed as follows. The motion sequence of the robot to climb the first step of the stair is shown in Fig. 6(a). First, let the robot stand in front of the stairs, and then swing the fore-legs to step on the first step sequentially. Next, coordinate all four legs to shift the robot body, so it inclines on the stair and parallels to the slope of the stairs. Then, swing the hind-legs to the right configuration preparing for the climbing. The configuration of the robot shown in the bottom figure of Figure 6(a) matches the configuration of steady-state climbing.

The motion sequence of the robot to transient from the last step of the stair to the top flat ground is shown in Fig. 6(b). When the robot reaches the last step of the stair, the fore-legs stretch to their maximum length to shift the COM of the robot pass the edge of the last step. Then, rotate the fore-legs to stably lie down the robot body on the ground. Next, coordinate all four legs to stand the robot up.

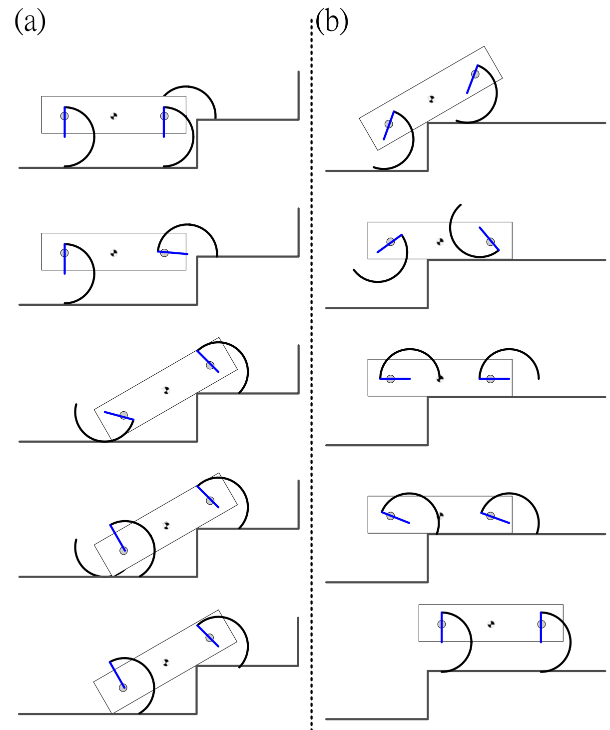

Fig. 6. (a) "first step" algorithm (b) "last step" algorithm

VI. Simulation and Experiment

The algorithm developed in the previous sections was simulated in Matlab with a particular set of parameters which matches the parameters of *Quattroped* [15] and of general local stairs (W=30, H=18). Figure 7(a) shows sequential snap shots of simulated result while the robot climbs one step. The movie of complete simulation is available as the supplemental material associated with this paper. The simulation shows that the legs can be coordinated to swing from the lower step to the upper one in sequences within one travel distance PL of COM without any interference, as expected from the analysis.

The algorithm was also implemented in the robot *Quattroped* and evaluated experimentally. Figure 7(b) shows the sequential images extracted from the video recording of robot climbing. The configurations of the robot in the 7 images also correspond to those of 7 simulation snap shots shown in Fig. 7(a). The full video is also available as the supplemental material associated with this paper. The video confirms that the algorithm is functional and the robot can indeed climb the stair.

VII. Conclusion

We report on the algorithm of trajectory planning for quasi-static stair climbing in the leg/wheel hybrid mobile robot. The detailed development is based on the workspaces of the legs and on the geometrical interactions between the robot legs and the stair. In addition, a brief study on the quasi-static stability of the robot is included and the

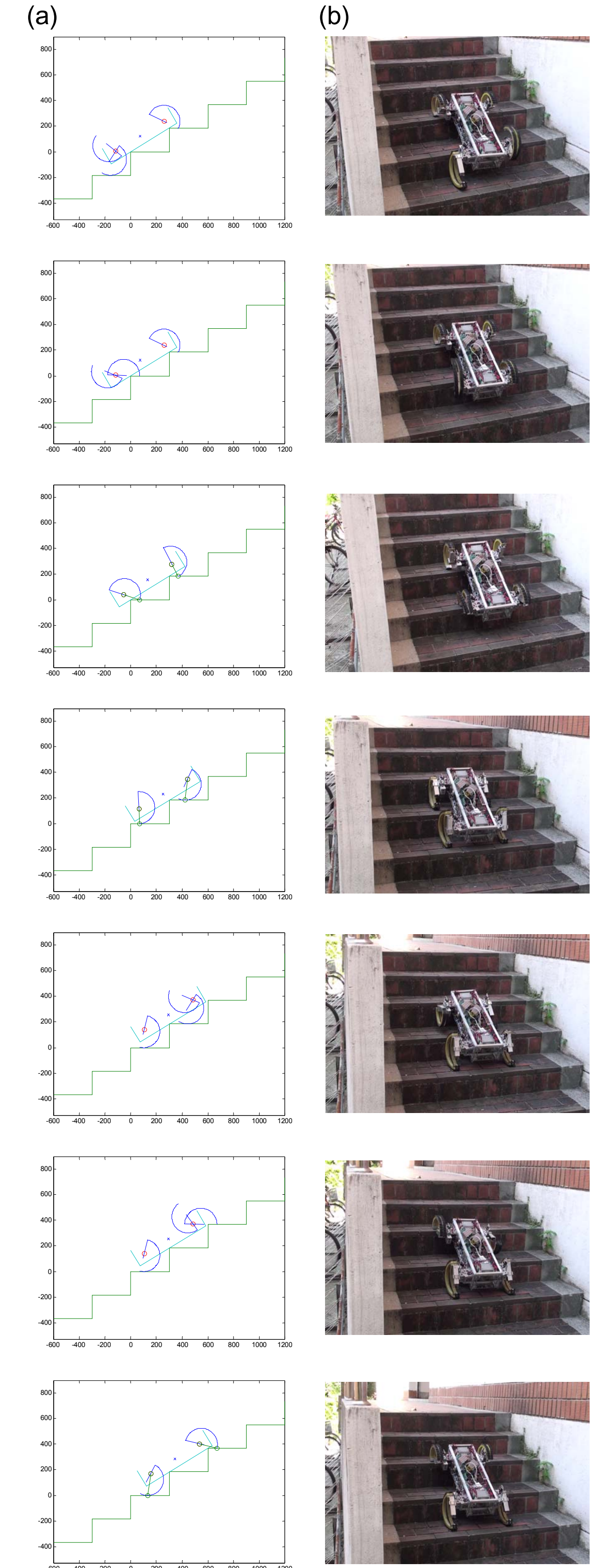

Fig. 7. The simulation (a) and the experiment (b) results of stair climbing in a quadruped by utilizing the proposed algorithm and trajectories.

associated concepts are also integrated in the proposed detailed trajectory planning. Finally, the algorithm is simulated and evaluated experimentally, which confirms the proposed algorithm is functional.

We are currently in the process of developing feedback mechanism of the algorithm, which will further tolerate a much wider geometrical variations of the stairs. In the meantime, the dynamics of the system is under investigation as well.

VIII. ACKNOWLEDGMENT

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